Efficient Secure AC OPF for Network Generation Capacity Assessment

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***Abstract*—This paper presents a novel method for determining the capacity of a network to accommodate new generation under network security constraints. The assessment is performed by maximizing the total generation capacity in an optimal power flow model; this is solved by gradually adding limited numbers of line outage contingencies, until a solution to the complete problem is obtained. The limit on the number of contingencies added is key to the method's efficiency, as it reduces the size of the optimization problems encountered. Moreover, varying this limit on contingencies added provides a simple and highly efficient means of searching for multiple local optima of the nonlinear optimization problem. The method has been tested on a modified version of the highly meshed IEEE Reliability Test System with *N*-1 security, where a significant reduction in the system's capacity for new generation is seen when security constraints are imposed. The method is generic and may be applied at any voltage level, for other security models and for other similarly structured problems such as the analysis of multiple resource availability scenarios.**

***Index Terms*—Load flow analysis, optimization methods, power generation planning.**

I. Introduction

WITH the current drive toward renewable and other low-carbon generation, the geographical pattern of generator locations is changing. As a result, there is now significant penetration of generation in parts of the network, particularly the distribution network, where formerly there was mainly load. A range of technical impacts (e.g., voltage rise, reverse power flows) dictate the amount of generation that may be connected without resort to network reinforcement. To maximize the potential of a network to support such distributed generation, it is important that these factors are carefully assessed, as poorly placed generation can significantly reduce the total potential for connecting generation [1]. Effective and efficient methods of assessing the capacity of the network to accommodate generation are therefore important, and there is a need to incorporate as many of the relevant technical constraints as possible, including security considerations.

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Several authors have proposed mathematical optimization-based approaches to network-wide planning of generation capacity and location, as opposed to considering one-generation site at a time. These have included a linear programming model to determine the optimal allocation of generation [2], tabu search in a loss minimization problem [3], and the use of a genetic algorithm to solve a multiobjective problem, considering losses, costs, and power generated [4]. In addition, earlier work involving two of the present authors has demonstrated the use of an ac optimal power flow (OPF) model to assess a network's capacity to accommodate generation [1], [5]. That framework forms the basis for this study.

The OPF was originally developed in the 1960s for network-constrained economic dispatch and has since been applied to many other problems [6], [7]. Its use in assessing the capacity of networks for connecting generation differs from the economic dispatch OPF both in the objective function (maximum network generation capacity, as opposed to minimum operating cost) and the decision variables (the capacity of potential generators, as opposed to the output of fixed-capacity generators). In addition, for economic dispatch the fixed demand limits the degree of network congestion. Here, the maximization of capacity, with surplus generation above the local demand being exported to an external grid, brings about greater levels of congestion; in a sense, the question is "how hard can the network be run?"

All transmission networks and many distribution networks are designed to operate in a secure mode to ensure continuity of supply under an outage (or contingency) of a circuit (*N*-1 security), or in some cases any two circuits (*N*-2). The constraints imposed by secure operation reduce the transfer capacity of the network, and specialized approaches have been developed to solve the resulting large security-constrained OPF models (SCOPF). One common option is to preselect a limited number of outage contingencies, which are likely to be significant [6]. An alternative approach, which allows efficient consideration of all contingencies, is to build a solution to the full problem by solving a series of subproblems, in which appropriate combinations of contingencies are added at each iteration [8].

This paper presents a new efficient solution method for OPF models used to assess network generation capacity under security constraints. The method is demonstrated on a modified version of the meshed IEEE 73-bus Reliability Test System (RTS) [9] with *N*-1 security, in which new generators are given firm connections. The method is, however, generic and may be applied at any voltage level, for other security models, and for other similarly-structured problems such as the analysis of multiple availability/demand scenarios for renewable resource availability. This solution approach brings two major advances. First, it resolves the problem that greater network congestion is encountered in comparison to security-constrained operational cost minimization, while still ensuring convergence of the OPF algorithm. Second, it provides a means of searching for multiple locally optimal solutions to the OPF model while retaining the great efficiency benefits arising from the use of warm starts in classical optimization algorithms.

II. Optimization Model

The OPF model for generation capacity assessment without security constraints is discussed in detail in [1]. The mathematical structure of the constraints is very similar to more familiar OPF application of cost minimization. The principal and fundamental difference lies in the objective, which is to the total generation capacity in the network:

$$\max \sum_{n \in N} p_n \tag{1}$$

where N is the set of possible locations for new generation, and p_n is the MW generation capacity allocated to site n; if generation exceeds local demand, the excess is exported to an external network. Using a continuous variable for generation capacity at each site is appropriate for a variety of distributed sources, where individual generator unit ratings might only be a few MW.

This formulation implicitly assumes firm connections. At distribution level, this correctly models the common situation in which the network operator is unable to dispatch generation. At transmission level, where generation may be constrained at cost to the system, this model will show the absolute potential of network sections for new generation (in a complete transmission system, where substantial exports are not possible, generation and load must be balanced at all times; in this case an assessment of potential for new generation is not relevant without market considerations.)

Modifying this formulation to perform a cost benefit analysis, including the cost of existing generation, a simple network upgrade model [5], and possibly the cost of additional interconnection to other networks, is relatively straightforward; the solution technique, which is the main point of this paper, would be unchanged. As discussed in Section III, the degree of congestion in generation maximization is higher than in the cost minimization; as a consequence, even if there are other terms present, rewarding increased generating capacity in the objective function requires a modified solution approach.

The control variables in the optimization model are the new generator capacities, which are the engineering decisions made by the model. The other decision variables in the optimization problem are state variables. Furthermore, features such as Var sources and tap changing transformers may be included using standard power flow equations [10] without changing the solution technique. The intact-network constraints are standard power flow equations, apart from the presence of new generators; these add an extra power injection term in the Kirchhoff current law constraints.

For the security model, constraints are added to represent the power flow equations for each contingency (i.e., circuit outage) considered. The reference bus is located at an external connection. In the intact network power flow equations, all external connections are equivalent (in the nonsecurity-constrained OPF, the only special feature of the reference bus is its role as the reference for voltage phase; the voltage levels at all external connections, along with the voltage phase at connections other than the reference bus, are decision variables.) The reference bus is the slack bus in the contingency power flow equations, in which it is a (V, δ) bus. Any other external connections are modeled as (P, Q) buses in the contingency flows. Where there is just one external connection, this may still have multiple circuits for security purposes, but the detail of these connections is not modeled.

New generation and load buses are also modeled as (P, Q) nodes, while any existing voltage-controlling generators are (P, V) nodes. The new generators are run in constant power factor mode, as is common with distributed generation [11], with the power factors of all the new generators equal. It is reasonably straightforward to use voltage or other control modes instead [12]. Thermal, voltage, and generation level constraints are included to ensure that the power flow remains feasible postcontingency; the emergency (postcontingency) voltage and flow limits may differ from their precontingency values. Postcontingency ramp rates are not explicitly taken into account in the contingency constraints, but the time required for system restoration will partly determine how much the emergency limits can be relaxed from the normal state ones.

A full mathematical formulation of the optimization model is given in the Appendix.

III. SCOPF Solution Method

A. Previous Approaches

In a secure dc OPF, the linearity of the problem allows individual contingency flow constraints to be included, without having to add the entire set of contingency power flow constraints. In the nonlinear ac OPF, however, limiting the postcontingency flow on one line when another suffers an outage requires the inclusion of the entire postcontingency power flow. Many approaches to the solution of large-scale ac SCOPFs have, therefore, involved the pre-selection of a small number of the most significant contingencies. Examples include [13], where the two most important contingencies in a very large system are chosen manually, and [14], where a sophisticated automatic selection is performed.

The methodology presented here develops that of Alsac and Stott [8] to solve the generation maximization problem efficiently. They used an iterative process where first the nonsecure OPF is solved, and then all contingencies in which power flow constraint violations occur are added to the security model. The resulting SCOPF is then solved, and the process repeated until no violations are found. As mentioned earlier, in generation maximization the level of congestion is much greater, because generation is limited only by the network constraints rather than by demand. As a result, when the base case OPF is solved, many (indeed possibly all) of the contingencies will show network constraint violations, using the original Alsac-Stott algorithm developed for cost minimization, these would all be added to the OPF model. Direct solution of the resulting large optimization models requires substantial computer time.

This paper shows that great computational benefits can be obtained, while still guaranteeing convergence of the algorithm, by limiting the number of contingencies added at each stage (and

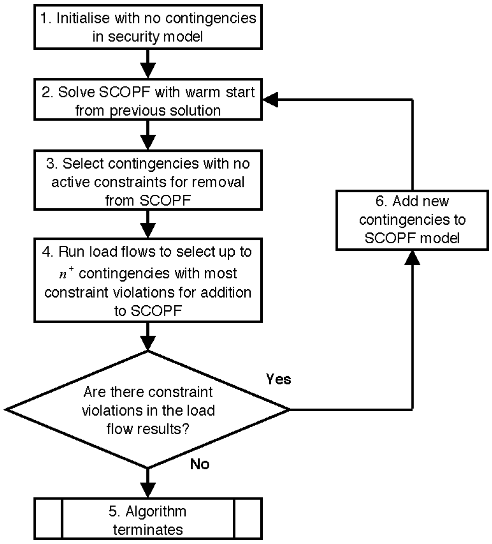

Fig. 1. Solution algorithm for the security constrained OPF.

hence size of the OPF models solved). It will also be seen that, where this is necessary, it is possible to search efficiently for multiple local solutions by varying this limit on contingencies.

A related approach to solving SCOPF problems has been proposed independently [15] and applied to cost minimization. As demonstrated for the first time here, this class of approach is of particular importance in applications such as generation maximization where flows are not limited by nodal demands. Moreover, it will be shown that the varying the limit on contingencies added provides a highly efficient means of searching multiple local optima of the nonlinear optimization problem, where warm starts are critical to the efficiency of a solution method, a simple multiple-start approach may be very time consuming.

B. Solution Methodology

The proposed solution method for the SCOPF is as follows (see also Fig. 1). M is the set of all contingencies considered, M^{sm} is the set of contingencies explicitly included in the SCOPF, $M^{+(-)}$ is the contingencies to be added to (removed from) M^{sm}, and M^{nr} is the contingencies, which have never been removed from M^{sm}.

1) Initialize the security model with $M^{\mathrm{sm}} = \emptyset$ (no contingencies initially in security model). Initialize $M^{\mathrm{nr}} = M$ (no contingencies yet removed from security model).
2) Solve SCOPF with contingencies M^{sm}. For contingencies, which were included in the previous SCOPF, warm start the contingency variables from their values in the previous solution. For contingencies not in the previous OPF, warm start the contingency variables from the previous base case solution.
3) Define M^- to be all contingencies in $M^{\mathrm{sm}} \cap M^{\mathrm{nr}}$ with no active voltage, reactive power, or flow limit constraints.
4) Run contingency load flows for all contingencies in $M \setminus M^{\mathrm{sm}}$, i.e., those not considered in the SCOPF.
   a) If more than n^+ contingencies give constraint violations, define the set M^+ to be the n^+ contingencies, whose load flows give the most constraint violations.
   b) Otherwise, define M^+ to be all contingencies giving constraint violations.
5) Terminate algorithm if no constraints are violated in these load flows.
6) Update the list of contingencies in the SCOPF, M^{sm}.
   a) Add the contingencies in M^+ to the SCOPF, i.e., in set notation update $M^{\mathrm{sm}} = M^{\mathrm{sm}} \cup M^+$.
   b) Remove the contingencies in M^- from the SCOPF and also from $M^{\mathrm{nr}} = M$ (the set of contingencies which have never been removed), i.e., update $M^{\mathrm{sm}} = M^{\mathrm{sm}} \setminus M^-$ and $M^{\mathrm{nr}} = M^{\mathrm{nr}} \setminus M^-$.
7) Go to Step 2.

Any constraint violations detected in load flow runs (i.e., in contingencies not included in the most recent SCOPF model) will be eliminated in subsequent SCOPF solutions, which explicitly consider greater numbers of contingencies. When the algorithm terminates, it does so at a local minimum of the SCOPF, including warm starting from all contingencies, not just those explicitly included in the security model M^{sm}.

This algorithm includes three augmentations beyond the simplest possible implementation of Alsac and Stott's approach:

1) Warm Starting From the Previous Solution: It is to be expected that appropriate warm starts will accelerate the solution. Here, the intact network variables from the previous SCOPF solution are used as the starting values of new contingency variables. Using contingency variables from the previous solver run for the warm start requires slightly more complex coding and gave no consistent benefit in run time.

2) Limit on the Number of Contingencies Added: The Alsac-Stott method may add a very large number of contingencies to the security model on the first iteration. Moreover, explicitly including the most severe contingencies in the security model may also eliminate violations in other contingencies, which are not explicitly considered (the former are sometimes known as umbrella contingencies [16].) It is, therefore, more efficient to limit the number of contingencies added on each iteration to the worst n^+ in terms of constraint violations.

3) Removal of Contingencies From the Security Model: The removal of contingencies in Step 6 attempts to identify those whose explicit consideration is unnecessary; this takes the idea behind restricting the number added a step further. The size of the OPFs solved is thereby reduced, leading to a corresponding reduction in the time taken per OPF solution. The restriction that a contingency may only be removed once from the security model is necessary to ensure eventual termination of the algorithm, as eventually all contingencies must be added if termination has not occurred. This prevents the algorithm from cycling, alternately adding and removing the same contingency. However, for the test cases run here, the benefit in time per OPF solution is more than cancelled out by the requirement for a greater number of OPF runs. Detailed results including contingency removal are, therefore, not presented in this paper; it would be interesting to investigate whether contingency removal is beneficial on other networks or classes of problem.

C. Implementation

The model is coded in the AIMMS optimization modelling environment [17]. In addition to OPF models, AIMMS can also be used to run the necessary load flow problems (this is achieved by formulating an optimization problems with a constant objective function. The optimization solver then looks for feasibility alone and acts as a nonlinear equation solver.) Use of a modeling language reduces development times, because the model structure may be specified as on paper, with the data being stored separately. AIMMS then generates the specific instance of the optimization problem from the model structure and data and passes it to the solver, also providing automatic generation of first and second derivatives of the constraint functions when required.

The AIMMS may be linked to a selection of efficient commercial solvers; here, the CONOPT generalized reduced gradient solver [17] is used. With default settings, it proved to be competitive with the KNITRO interior point solver [17] in terms of speed and was absolutely reliable in convergence for the SCOPFs in this paper. While KNITRO was slightly faster than CONOPT when it did converge, using standard settings it frequently failed to do so when the number of contingencies included exceeded low single figures. Both solvers are able to handle large numbers of hard nonlinear equality and inequality constraints automatically and exactly.

Occasionally, load flows did fail to converge in CONOPT on early iterations of the algorithm. However, as long as these do not occur later on, this does not risk the method terminating unsuccessfully. It is likely that if the load flows were implemented using specialized code, they would be faster and more robust, but this would be at the expense of greatly increased development time.

IV. Case Study

A. Test Problem

To demonstrate the solution method, a modified version of the IEEE 73-bus RTS is used. It consists of three identical 24-bus networks and interconnections between them. The RTS layout is shown schematically in Fig. 2, along with a more detailed plan of area 1. It is not intended to be representative of any particular power system, but it provides a convenient, reasonably large-scale heavily meshed network on which to demonstrate the SCOPF solution method. This test problem shows the method's efficiency on large problems with many line outage contingencies, but the method is generic across different voltage levels and security models.

A total of 15 new generation units are allowed to be connected at buses 1, 2, 7, 15, and 22 in each area of the RTS. The line characteristics of the RTS remain the same but the original demand specified was halved (to 4.22 GW) to ensure a feasible problem. In real-world applications, the algorithm would typically be run for the "worst case" scenario, which for an exporting network would be maximum generation and minimum load.

One hundred and five contingencies are considered in the assessment including all single-line outages except lines 10, 11, 16, 17, and 23 in each area. Including any of these would leave an infeasible problem; lines 211 and 311 are single-circuit radial connections.

In order to demonstrate the flexibility of the proposed method, calculations are performed with either one or four external interconnections, and one or three existing voltage-controlled generators in each area of the RTS (these are not the same as the generators in the original RTS specification; only the original layout and line properties are used.) The four cases are described in Table I. All external interconnections are modeled as having unlimited capacity (with finite capacities the solution technique would be unchanged), and the reference bus is always located

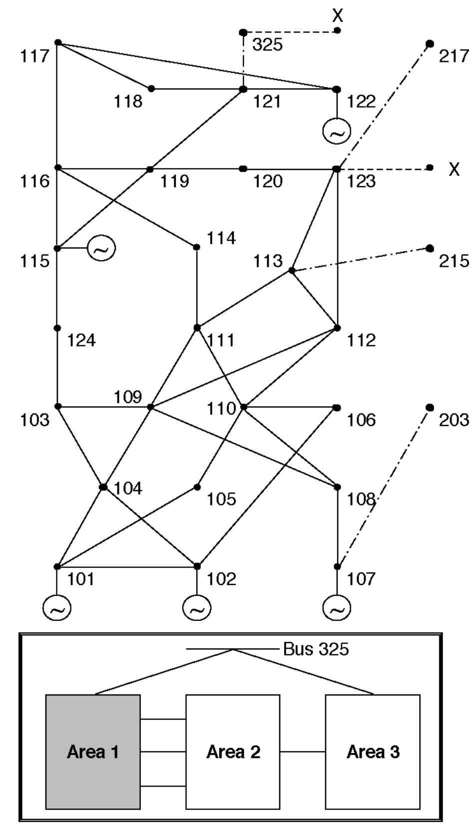

Fig. 2. Lower panel: schematic representation of the IEEE 73-bus RTS including interconnections; each of the three identical areas contains 24 buses. Upper panel: area 1 of the RTS, dashed lines to X's indicate connections to external connections, dot-dashed lines indicate connections to other areas of the RTS, and new generator sites are marked with the conventional generator symbol.

TABLE I
External Interconnections and Existing Generator Locations for the Four Network Cases. $sxgy$ Denotes the Case With x External Connections and y Existing Generators

Case	Interconnections	Existing Generators
s4g3	325,123,223,323	118,218,318
s1g3	325	118,218,318
s4g9	325,123,223,323	113,115,118,213,215 218,313,315,318
s1g9	325	113,115,118,213,215 218,313,315,318

at bus 325. In the cases with three existing voltage-controlling generators, each has real power output of 800 MW and reactive power limits of ± 400 MVar; where there are nine generators, each has real power output of 200 MW and reactive capability of ± 100 MVar.

B. Results

The OPF model was run for each case, both with and without security constraints. The optimal new generation capacities available in the network are shown in Table II.

The results without security constraints show that for the two cases with a single interconnection, the connectable capacities are very similar. There is slightly more capacity available where

TABLE II
LOCALLY OPTIMAL SOLUTIONS FOR THE FOUR NETWORK CASES
(* INDICATES THAT MULTIPLE LOCALLY OPTIMAL SOLUTIONS WERE FOUND IN THESE CASES—SEE FIG. 3)

	Optimal capacity (GW)	
Case	Non-secure	Secure
s4g3	4.578	3.080
s1g3	3.094	2.595
s4g9	5.173	2.901*
s1g9	3.008	2.521*

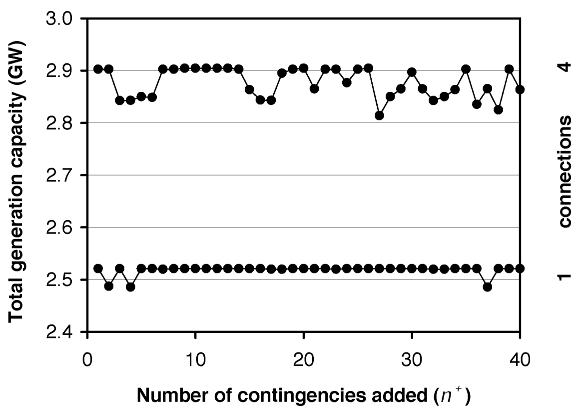

Fig. 3. Locally optimal solutions found for the cases with nine existing generators.

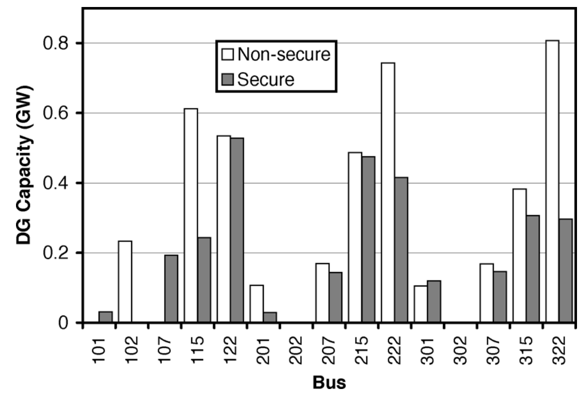

Fig. 4. Optimal generation site capacities with and without security constraints for case s4g3.

there is a greater number of voltage-controlling existing generators; it appears that despite a larger existing generating capacity (s1g3: 2400 MW; s1g9: 1800 MW) the greater overall reactive capability in s1g3 allows slightly more (86 MW) new generation to connect. The two cases with four export connections have greater connectable capacities, as the particular constraints affecting bus 325 are less important. The available capacity differs much more between these cases; this is explained by the difference in existing generating capacity (~600 MW).

As expected, in all cases there are distinct reductions in capacity when the security constraints are applied. These reductions range from 16% for the two cases with a single export connection (s1g3 and s1g9) up to 43% for case s4g9. Again there is a pattern in the capacities. The single connection cases each show secure capacities of around 2.5 GW. The multiconnection cases now show fairly similar secure capacities and hence different reductions relative to nonsecure conditions. The difference in existing generating capacity appears to play little part in these cases under secure conditions. The reason for this differing behavior between the nonsecure and secure cases is not clear; this issue demonstrates one benefit of using mathematical tools to analyze complex nonlinear problems, where a more heuristic approach might not be able to account for all relevant phenomena.

Although the results are entirely repeatable between runs, not all runs find precisely the same optimal capacity. As Fig. 3 shows, the cases with nine existing generators, and particularly those with multiple network interconnections, have multiple locally optimal solutions depending on the number of contingencies added.

For the cases with just one existing generator in each area, just one locally optimal solution is found; however, for cases s1g9 and s4g9 multiple locally optimal solutions were found depending on the limit on contingencies added, as shown in Fig. 3. It would appear that when there is more than one voltage-controlled generator that produces or consumes reactive power in each area, and particularly when there are also four external connections through which power may be exported, the greater flexibility in the system results in these multiple locally optimal solutions. The difference in capacity between different locally optimal solutions is fairly small: for the single connection case, the worst solution differs from the best by around 1.4%, while for the multiconnection case, the worst solution is around 5% below the best.

It is also informative to examine capacities at individual locations (the layout of area 1 of the RTS is shown in Fig. 2.) Fig. 4 shows the individual site capacities with and without security constraints applied for s4g3. It is immediately apparent that in most cases, when *N*-1 security is introduced, the capacities do not scale equally. For instance, the capacity at bus 322 decreases by 63%, while that at bus 122 barely changes. Most of the generation is sited at buses 15 and 22 in each area of the reliability test system, which are those closest to the external connections. Where there are significant differences between the three areas of the RTS, this is partly explained by the pattern of interconnections between areas. For example:

1) When security constraints are imposed, considerable generation capacity transfers from bus 102 to 107. Bus 107 has an interconnection to area 2, so it appears to be a robust site for generation under *N*-1 security. Buses 207 and 307 do not have a similar interconnection, and no similar transfer of capacity is seen in areas 2 and 3.
2) The main loss of capacity between the nonsecure and secure models is at buses 115, 222, and 322. Unlike bus 115, the "equivalent" buses 215 and 315 are near interconnections to areas 1 and 2, respectively, making them robust sites under security constraints. Similarly, bus 122 is near the interconnection from 121 to 325, whereas there is no similar interconnection near 222 or 322.

Once more, the finer detail of the results is determined by the subtle interplay between the various voltage and thermal constraints in the SCOPF model.

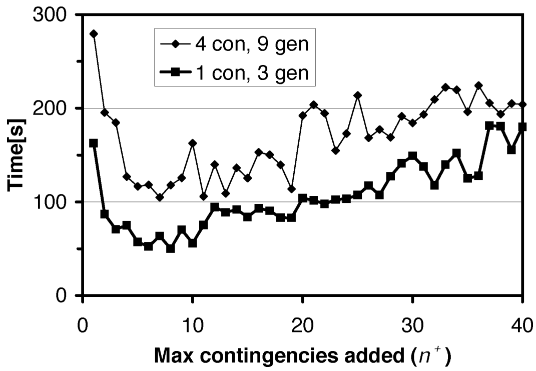

Fig. 5. Time taken to solve to local optimality for network cases s1g3 and s4g9, for a range of limits on contingencies added. The other two cases gave similar plots.

C. Performance of Algorithm

This section compares the performance of the algorithm with more conventional approaches. As timings of runs in AIMMS (running under Windows XP with an Intel 2.13-GHz dual core processor and 2 GB of RAM) varied slightly from run to run, the times given are the *smallest* of three runs, to reflect most accurately the actual processor time used; for the three runs the actual calculations performed were identical, but the time taken for completion could vary substantially between runs because of other active processes on the PC. The calculation results were repeatable between runs. The memory usage for the very largest models was around 550 MB; this is smaller than the physical memory of any modern PC.

Fig. 5 shows the solution times for cases s1g3 and s4g9, plotted against the maximum number of contingencies added per iteration n^+. The shortest times were around 50 and 100 s, respectively. For comparison, direct solutions of the various cases were performed using flat starts [i.e., all variables initially zero except the voltage levels, which were set to 1 per unit (p.u.)]. The resulting mathematical programs had around 100 000 variables and constraints. In cases s4g3 and s1g3, for which only one locally optimal solution was found, the solution found by this direct method was the same as that found by the approach presented in this paper; this demonstrates the validity of the new method. For case s4g3, the direct solution with flat starts took around 11 300 s; direct solution warm starting from the base case (without security constraints) OPF solution reduced execution time to 513 s.

The effectiveness of the enhanced features of the algorithm described in Section III-B is discussed in the next paragraphs.

1) Warm Starts: As expected, using warm starts accelerates convergence of the algorithm. Run times using flat starts are not plotted, as in all calculations performed they were at least three times greater than the warm start run times.

2) Limit on the Number of Contingencies Added: As seen in Fig. 5, the smallest run times occur when the limit n^+ on the number of contingencies added is between about 5 and 20.

With no limit on the number of lines added at each stage, the run times were 194 s without removal and 240 s with removal for network case s4g3 (33 contingencies are then added on the first iteration.) In the other cases, without the n^+ limit almost all of the contingencies are added in the first stage, and the algorithm then terminates as there are no violations in the remaining contingencies. The run times are then 458 (case s1g3), 908 (s4g9), and 550 s (s1g9). By comparison with Fig. 5, it is clear that imposing the limit on contingencies added is beneficial in all cases.

The total time taken is a tradeoff between the time for each OPF solution (smaller at low n^+ due to both the smaller optimization problems and the better warm starts) and the number of iterations (which as expected is smaller at large n^+). With the best times for this network occurring for n^+ of between 5 and 20, the lower end of this range is probably a good starting point for single runs on other problems. Choosing too big a value for n^+ carries a risk of large, hard mathematical programs being encountered; adding fewer than five contingencies is likely to result in long run times due to the increased number of iterations.

V. Discussion

The method offers an efficient means of determining the network capacity available for generation connection, for a given network configuration and loading condition, and with consideration of security constraints. It would also provide an effective means of assessing single sites as it offers an automated means of determining whether a proposed connection exceeds the capacity of the network without the need for extensive manual examination of multiple scenarios. Furthermore, as demonstrated in [1], there is also the option to use it to examine the impact of planning or connection decisions on future network capacity.

In a monopoly utility, this approach could be used directly in decisions on generator locations. In a liberalized market, where ownership of the network and generators are separate, it could find application within the process of determining use-of-system charges; within such a framework, the method could indicate good and bad places for generators to connect.

The structure of the method is such that it can be used not only in the large meshed network shown here, but also where a distribution network is run in radial mode but security of supply is maintained using network reconfiguration. The method assumes that generation capacity is firm, but reliability and variability of generators could be taken into account using capacity factors such as those defined by the U.K. Energy Networks Association [18]. Further work is planned on both of these aspects.

The method can also be used for other contingency types, such as generator trips [19] and network reconfiguration. One key point is whether a very high proportion of contingencies in any one category restricts the optimal solution; if this is the case, it might be most efficient to force their addition to the model early on in the solution process.

In addition to the substantial efficiency benefits it brings for a single run, the ability to vary the limit on contingencies added can bring greater benefits still in problems with multiple local solutions. The solution found using the "sequential warm start" method is necessarily a local solution of the SCOPF including all contingencies. However, due to the nonconvexity of the ac OPF, there is no way of proving that the global solution has been found [20], even when runs of the algorithm with different n^+ have produced just one solution (although it is likely that the single solution is then the global one).

For large nonconvex nonlinear optimization models such as this, there are no efficient general purpose global solution methods available. In practice, therefore, it is typically best to perform multiple runs of the same problem with different starting points. In a sense, this has already been done by using different values of n^+. For network case s4g9, where several different local minima were found, the best and most common solution is probably the global optimum. If multiple starts of the same process are required, this is complicated by the criticality of warm starts to the efficiency of the algorithm (possibly rendering impractical the simplest option of choosing random starts with initial values for variables chosen from their whole range). The ability to choose different n^+ in the same algorithm, therefore, provides a simple means of performing multiple starts, which also resolves the issue of which n^+ gives the quickest run time and best solution for an unseen problem. This benefit would apply in any SCOPF problem and not just generation maximization ones with an enhanced degree of network congestion.

In a distribution network, it is relatively unlikely that very extensive meshing, or significant numbers of voltage-controlling generators or interconnections will be encountered. It appears more likely then that a single local optimum will be found. In any case, in many planning applications, it is not absolutely necessary to know for certain that the best local optimum found is the global optimum; a technique is valuable if it finds a solution, which is better than those obtained by other means, and any good local optimum may, therefore, suffice. This might not be the case, however, where an ac OPF is given statutory authority, e.g., if it is used for generator dispatch in a pool system.

A similar issue arises where two different local solutions have similar objective function values. In this case, the difference in solution quality may well be within the approximation error in the model formulation, while the optimal values of decision variables are very different. This situation has indeed been seen in the more complex network cases run on the RTS. For instance, in the case s4g9, when the limit on contingencies added (n^+) changes from 8 to 9, the locally optimal objective changes by 0.63%; however, the average change in optimal generator capacities in area 1 is then 13.3%. Once more, if different local solutions would influence contractual decisions, then this is a particular cause for concern. On the other hand, under some circumstances it might be regarded as beneficial to have a range of good locally optimal solutions with similar objective function values, with a final decision being made on other grounds; where multiple local optima exist, this method provides a way of finding these multiple options.

All results here are based on the CONOPT 3.14A solver; it is possible that that contingency removal may still prove useful when working with other solution techniques (e.g., interior point) or indeed on other networks. If a different solver is to be used, it must be remembered that while they can be highly efficient, interior point methods are generally so not well suited to warm starts [21], as CONOPT's generalized reduced gradient method. As demonstrated here, complications arising from the presence of large numbers of nonlinear equality constraints are expected to be due to multiple local solutions, rather than any features of the solution method such as warm starting.

This work is currently being extended to problems involving nonfirm access with generation curtailment and multiperiod calculations including consideration of variability of renewable resources [22]. In such applications, the number of scenarios considered grows exponentially with the number of renewable resource profiles. Direct solutions on realistic network models could, therefore, involve extremely large optimization problems, and the benefit from the method presented here is, therefore, expected to be considerable.

VI. Conclusion

This paper presents a method for determining the capacity of a network to accommodate generation under security constraints, making use of an OPF model designed to maximize generation capacities. The maximization of capacity, as opposed to cost minimization, brings about greater levels of congestion as power transfers in the network are not limited by fixed demand. For this application, therefore, specialized solution approaches are especially valuable.

The model is solved by gradually adding limited numbers of line outage contingencies to the model, until a solution to the full problem, including all contingencies, is obtained. The limit on the number of contingencies added is key to the efficiency of the method, as it reduces both the size of the optimization problems encountered and the difference between successive problems. Moreover, varying the limit on contingencies added provides a highly efficient way of searching for multiple locally optimal solutions of the nonlinear optimization problem.

The method has been tested on a modified version of the highly meshed IEEE RTS with *N*-1 security. When security constraints are imposed, there is a large reduction in the network's capacity for new generation, which emphasizes the importance of considering all relevant physical and operational constraints in assessments. The method is generic and may be applied at any voltage level, for other security models and for similarly structured problems including the analysis of multiple scenarios for renewable resource availability.

Appendix
OPF Formulation

A complete specification of the SCOPF model is given here.

A. Nomenclature

1) Base Case OPF: Sets

B	Set of buses (indexed by b).
L	Set of lines (indexed by l).
G	Set of existing generators (indexed by g).
N	Set of new generators (indexed by n).
X	Set of external sources (indexed by x).
G_b	Set of generators connected to bus b.

Parameters

$d_b^{(P,Q)}$	(P,Q) demand at bus b.
$V_b^{(+,-)}$	(max/min) voltage at b.
b_0	Reference bus.
$(p,q)_g^{(+,-)}$	(max/min) (P,Q) output of existing generator g.
β_g	Location of g, etc.
$p_n^{(+,-)}$	(max/min) capacity of new generator n.

ϕ Power angle of new generators.

f_l^+ Maximum MVA (S) flow on line l.

Variables

(V_b, δ_b) Voltage (level, phase) at b.

$(p,q)_g$ (P,Q) output of g.

p_n Real power capacity of n.

$(p,q)_x^X$ (P,Q) supplied by x.

$f_l^{(1,2),(P,Q)}$ (P,Q) injection onto l at (start, end) bus.

2) Security Model: Sets

M Set of contingencies (indexed by m).

L_m Set of lines available in contingency m.

Parameters

$V_b^{C,(+,-)}$ (max/min) voltage at b in contingency flows.

χ Increase in maximum flows postcontingency.

Variables

$V_{m,b}$ Voltage at b in contingency m.

[etc.]

B. OPF Without Security Constraints

1) Objective Function: The goal is to maximize the total capacity of the new generators

$$\max \sum_{n\in N} p_n \tag{2}$$

2) Capacity Constraint for New Generators:

$$p_n^- \le p_n \le p_n^+ \qquad \forall\, n\in N \tag{3}$$

3) Generation Level Constraint for Existing Generators:

$$(p,q)_g^- \le (p,q)_g \le (p,q)_g^+ \qquad \forall g\in G \tag{4}$$

4) Supply Level Constraint for External Sources:

$$(p,q)_x^{X,-} \le (p,q)_x^X \le (p,q)_x^{X,+} \qquad \forall\, x\in X \tag{5}$$

5) Voltage Level Constraint:

$$V_b^- \le V_b \le V_b^+ \qquad \forall\, b\in B \tag{6}$$

6) Reference Bus: Voltage angle is zero

$$\delta_{b_0} = 0 \tag{7}$$

7) Kirchhoff Current Law: $\forall b\in B$

$$\sum_{l\in L} p_b^{\mathrm{L}} + d_b^P = \sum_{g\in G_b} p_g + \sum_{x\in X_b} p_x^X + \sum_{n\in N_b} p_n \tag{8}$$

$$\sum_{l\in L} q_b^{\mathrm{L}} + d_b^Q = \sum_{g\in G_b} q_g + \sum_{x\in X_b} q_x^X + \sum_{n\in N_b} (\tan\phi) p_n. \tag{9}$$

Here, $(p,q)_b^L$ is total power injection onto lines at b. The reactive power line injections include the shunt capacitance term.

8) Kirchhoff Voltage Law (KVL):

$$f_l^{(1,2),(P,Q)} = f_{l,(1,2)}^{\mathrm{KVL(P,Q)}}(\mathbf{V},\boldsymbol{\delta}) \qquad \forall\, l\in L. \tag{10}$$

Here, $f_{l,(1,2)}^{\mathrm{KVLP}}(\mathbf{V},\boldsymbol{\delta})$ and $f_{l,(1,2)}^{\mathrm{KVLQ}}(\mathbf{V},\boldsymbol{\delta})$ are the standard Kirchhoff voltage law expressions for the power injections onto lines at the two terminal buses (denoted 1 and 2).

9) Flow Constraints at Each End of Lines:

$$\left(f_l^{(1,2),P}\right)^2 + \left(f_l^{(1,2),Q}\right)^2 \le \left(f_l^+\right)^2 \qquad \forall\, l\in L. \tag{11}$$

C. Security Model

The following constraints are added for all contingencies explicitly included in the security model, i.e., $\forall\, m\in M^{\mathrm{sm}}$.

1) Supply Level Constraint for External Connections:

$$(p,q)_x^{X,-} \le (p,q)_{m,x}^X \le (p,q)_x^{X,+} \qquad \forall\, x\in X\setminus\{b_0\} \tag{12}$$

2) Voltage Level Constraint:

$$V_b^{C,-} \le V_{m,b} \le V_b^{C,+} \qquad \forall\, b\in B \tag{13}$$

3) Reference Bus Constraints:

$$\delta_{m,b_0} = 0 \tag{14}$$

$$V_{m,b_0} = V_{b_0}. \tag{15}$$

The Reference bus is a (V,δ) bus in the contingency flows.

4) Existing Voltage-Controlled Generator Constraints:

$$\left.\begin{array}{c} V_{m,\beta_g} = V_{\beta_g} \\ q_g^- \le q_{m,g} \le q_g^+ \end{array}\right\} \qquad \forall g\in G. \tag{16}$$

5) Kirchhoff Voltage Law: Constraints take exactly the same form as (10), but for contingency m, a constraint expressing the power injections $f_{m,l}^{(1,2)}$ in terms of the contingency voltages $(V_{m,b}, \delta_{m,b})$ is only generated for the available lines $l\in L_m$. $V_{m,b} = 0$ for lines not in L_m.

6) Kirchhoff Current Law: $\forall\, b\in B$

$$\sum_{l\in L} p_{m,b}^{\mathrm{L}} + d_b^P = \sum_{g\in G_b} p_g + \sum_{x\in X_b} p_{m,x}^X + \sum_{n\in N_b} p_n \tag{17}$$

$$\sum_{l\in L} q^{\mathrm{L}}_{m,b} + d^{Q}_{b} = \sum_{g\in G_b} q_{m,g} + \sum_{x\in X_b} q^{X}_{m,x} + \sum_{n\in N_b} (\tan\phi) p_n. \quad (18)$$

7) Flow Constraints: The contingency flow limit may be raised above the base case by a factor χ:

$$\left(f^{(1,2),P}_{m,l}\right)^2 + \left(f^{(1,2),Q}_{m,l}\right)^2 \leq \left(\chi f^{+}_{l}\right)^2 \qquad \forall\, l \in L. \quad (19)$$

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